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Italiadomani  
PIANO NAZIONALE  
DI RIPRESA E RESILIENZA



Università  
degli Studi  
di Palermo

# Project FORESEEN

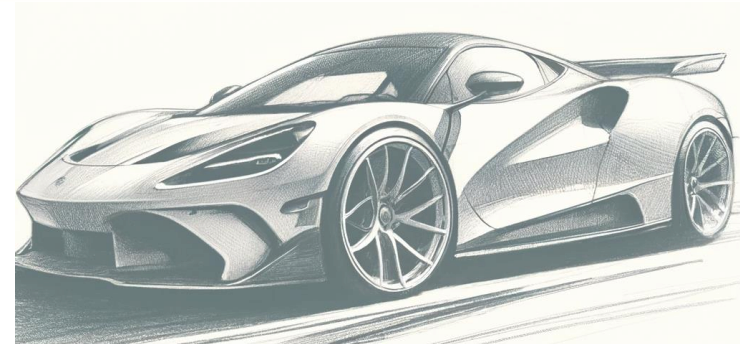
## Proposed Vehicle

### Models and Controllers

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Meeting of 15<sup>th</sup> April 2024

1. *Longitudinal Dynamics*
2. *Distributed Controller*
3. *Lateral Dynamics*



# M I R P A L A B

MOBILE & INTELLIGENT ROBOTS @ PANORMOUS LABORATORY

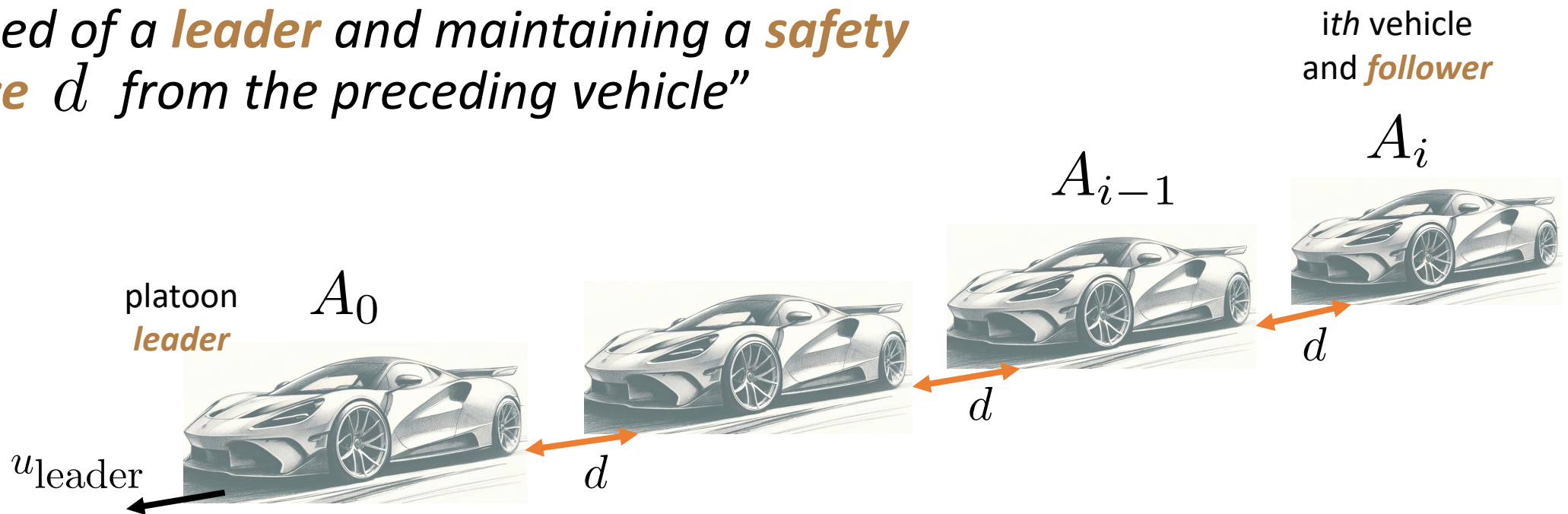


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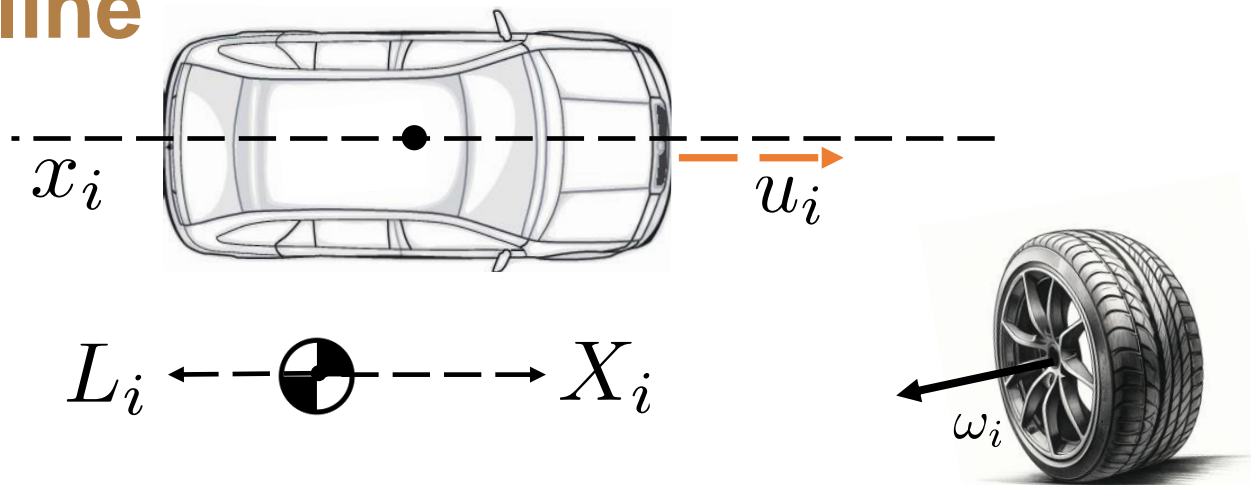


## The Platoon Scenario

- $N+1$  vehicles moving along a road in **platoon** configuration:  
“travelling as a string of vehicles while copying the speed of a **leader** and maintaining a **safety distance**  $d$  from the preceding vehicle”



# Longitudinal dynamics on straight line



$x_i$	position along line/curve	m
$u_i$	longitudinal speed	m/s
$\omega_i$	wheel rotation speed	rad/s
$m_i$	total mass	kg
$J_i$	wheel inertia	Kg m <sup>2</sup>
$R_i$	wheel radius	m
$X_i$	longitudinal force	N
$L_i$	(loss) resistance force	N
$T_i$	driving torque	Nm

$$\dot{x}_i = u_i$$

$$\dot{u}_i = \frac{1}{m_i} (X_i(u_i, \omega_i, t) - L_i(u_i))$$

$$\dot{\omega}_i = \frac{1}{J_i} (T_i - R_i X_i(u_i, \omega_i, t))$$

- $X_i(u_i, \omega_i, t)$  complex, almost unknown, depends on friction between road and (virtual) wheel tire
- $L_i$  includes aerodynamic drag, wind, bank angle effect

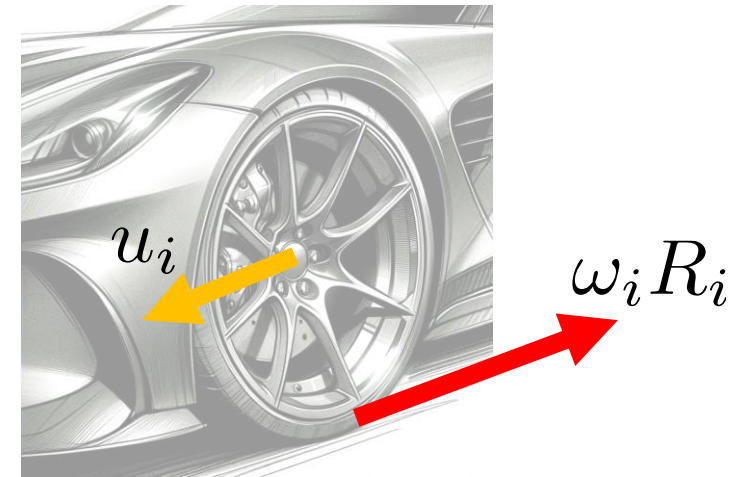
$$L_i(u_i) = 1/2 \rho C_{xi} S_{xi} u_i^2 + W_i + \dots$$





## Longitudinal Slip ratio & Force

$$\sigma_i = \begin{cases} \frac{\omega_i R_i - u_i}{\omega_i R_i} = 1 - \frac{u_i}{\omega_i R_i} & \text{if } \omega_i R_i > u_i \text{ (acceleration)} \\ \frac{\omega_i R_i - u_i}{u_i} = \frac{\omega_i R_i}{u_i} - 1 & \text{if } \omega_i R_i < u_i \text{ (braking)} \end{cases}$$



## Longitudinal Force Models

$$X_i = \mu_i Z_i$$

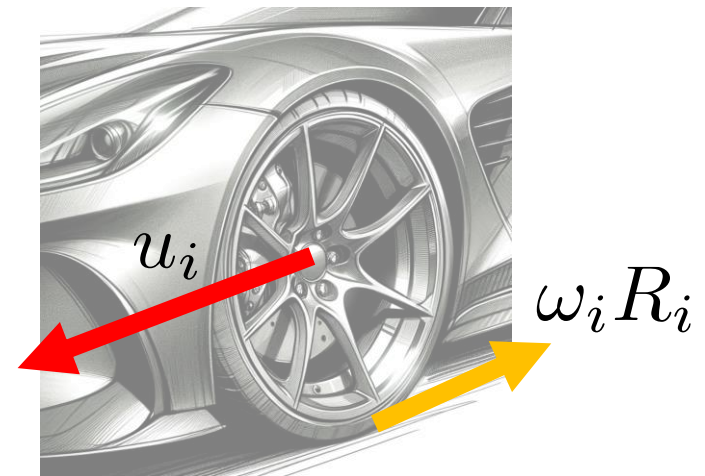
friction coeff.

$$Z_i \simeq m_i g \frac{a_{i,1}}{l_i}$$

vertical force

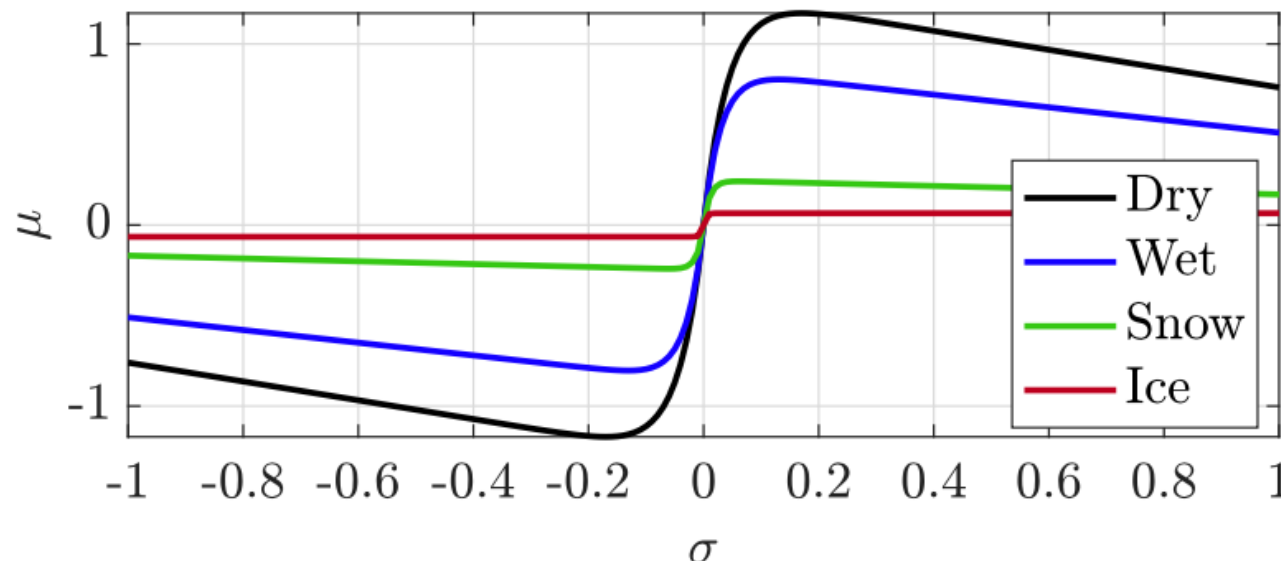
long. force

$$X_i(u_i, \omega_i, t) = \mu_i(\sigma_i, t) m_i g \frac{a_{i,1}}{l_i}$$



## Heuristics Friction Models

- Friction is a nonlinear function of *slip ratio* (state variable) and *Tire-Surface Interaction (TSI)* (external variable)
- TSI depends on the tire and road surface (e.g., dry, wet, snowy, icy)

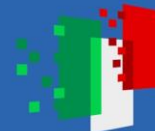


### • Pacejka's Magic Formula

$$\mu_i(\sigma_i, t) = \mu_{i1}(t) \sin(\mu_{i2}(t) \arctan(\mu_{i3}(t) \sigma_i + \mu_{i4}(t)(\mu_{i3}(t) \sigma_i - \arctan(\mu_{i3}(t) \sigma_i))))$$

### • Burckhardt's Model

$$\mu_i(\sigma_i, t) = \underbrace{\mu_{i1}(t)}_{\text{max value}} \left( 1 - e^{\underbrace{\mu_{i2}(t)}_{\text{curve shape}} |\sigma_i|} \right) - \underbrace{\mu_{i3}(t)}_{\text{value at full slip } |\sigma_i| = 1} |\sigma_i|$$



## Approximated Model for “small” slip ratio

$$\sigma_i = 0 \rightarrow \omega_i R_i = u_i \rightarrow \dot{\omega}_i R_i + \cancel{\omega_i \dot{R}_i} = \dot{u}_i$$

$$\dot{u}_i = \frac{1}{m_i} (X_i(u_i, \omega_i, t) - L_i(u_i))$$

$$R_i \dot{u}_i = \frac{1}{J_i} (T_i - R_i X_i(u_i, \omega_i, t))$$

$$\dot{u}_i = \frac{1}{m_i^*} \left( \frac{T_i}{R_i} - L_i(u_i) \right)$$

$$m_i^* = m_i + J_i / R_i^2$$

equivalent  
total mass

- *eliminates one equation*
- *independent of  $X_i$*
- driving torque appears in the speed equation





## Decentralized $i$ th controller (step 1: introduce a desired speed)

$$u_{id}(t) = u_{id}(u_{i-1}, u_i, t) \quad \leftarrow \text{ith desired speed}$$

$$\delta u_i = u_i - u_{id} \quad \leftarrow \text{ith speed tracking error}$$

$$\delta \dot{u}_i = \dot{u}_i - \dot{u}_{id} = \frac{1}{m_i^*} \left( \frac{T_i}{R_i} - L_i(u_i) \right) - \dot{u}_{id}$$

reference  
model

$$\delta \dot{u}_i + k \delta u_i = 0 \quad k > 0$$

$$\delta u_i(t) = e^{-\alpha t} \delta u_i(0)$$

$$T_i = m_i^* R_i (\dot{u}_{id} - k \delta u_i) + R_i L_i(u_i)$$

ith control input

- independent of  $X_i$



## Decentralized $i$ th controller (step 2: design the desired speed to maintain the safety distance)

$$\delta d_i = d_i - d \quad \leftarrow \text{ith distance error}$$

$$\begin{aligned} \delta \dot{d}_i &= \dot{x}_{i-1} - \dot{x}_i - \dot{d} = \\ &= u_{i-1} - u_i = u_{i-1} - u_{id} \end{aligned}$$

reference model

$$\begin{aligned} \delta \dot{d}_i + \alpha \delta d_i &= 0 \\ k &> 0 \end{aligned}$$

$$u_{id}(t) = u_{i-1} + \alpha \delta d_i = u_{i-1} + \alpha (x_{i-1} - x_i - d)$$

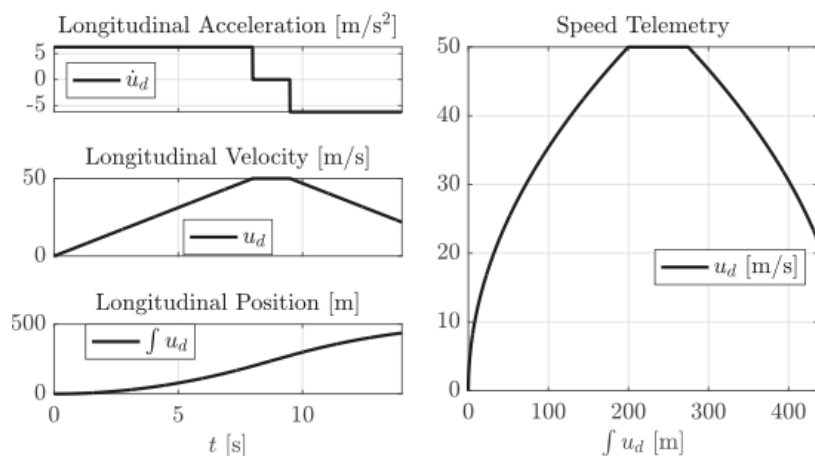
$i$ th desired speed

...it is extended to stabilize also slip ratio  $\delta \sigma_i = \sigma_i - \sigma_{id}$

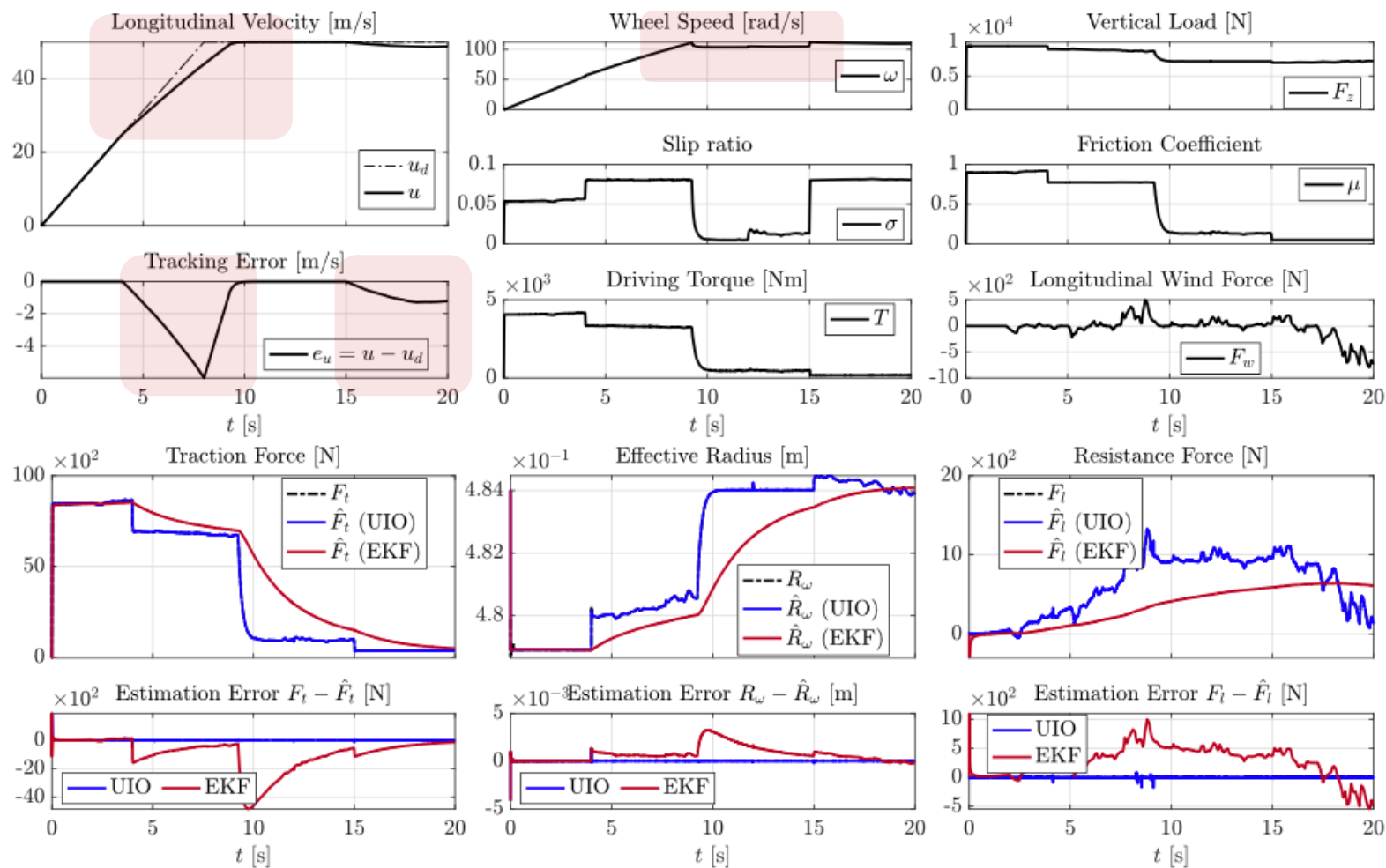




## Simulation on a single vehicle scenario w/ wind disturbance

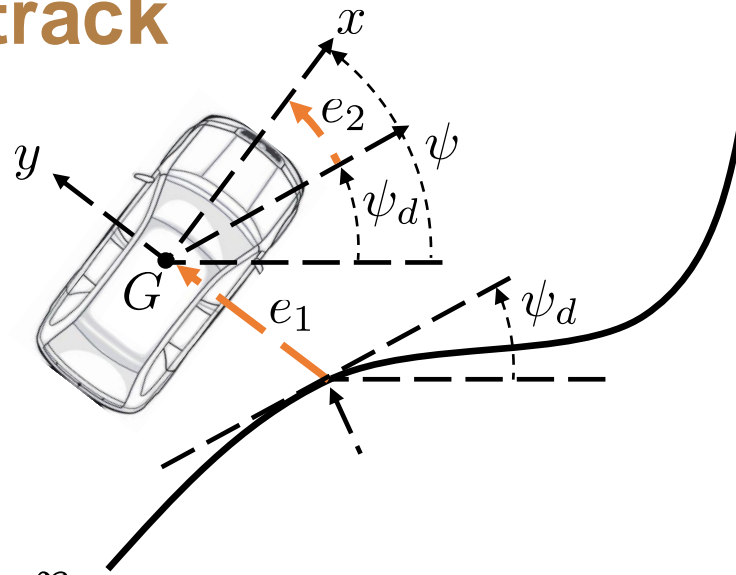


Road conditions switch from dry ( $t < 4$ ) to wet ( $4 < t < 12$ ) to snow ( $12 < t < 15$ ) and ice ( $t > 15$ ). Sudden wind gust at  $t = 2$ .





# Lateral dynamics single-track for small steering angles



$y_i$	lateral position w.r.t. path	m
$v_i$	lateral speed	m/s
$\psi_i$	c.c. orientation w.r.t. path	rad
$r_i$	angular speed about z	rad/s
$I_i$	vehicle inertia about z	Kg m <sup>2</sup>
$\delta_i$	steering angle	rad

$$\dot{y}_i = v_i$$

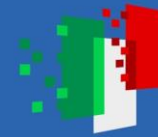
$$\dot{v}_i = -u_i r_i - \gamma_{i1} \frac{v_i}{u_i} + \gamma_{i2} \frac{r_i}{u_i} + \gamma_{i3} \delta_i$$

$$\dot{\psi}_i = r_i$$

$$\dot{r}_i = \gamma_{i4} \frac{v_i}{u_i} - \gamma_{i5} \frac{r_i}{u_i} + \gamma_{i6} \delta_i$$

- $\gamma_{ij}$  depend on **cornering** coefficients, vehicle mass and inertial
- non-null speed  $v_i \neq 0$





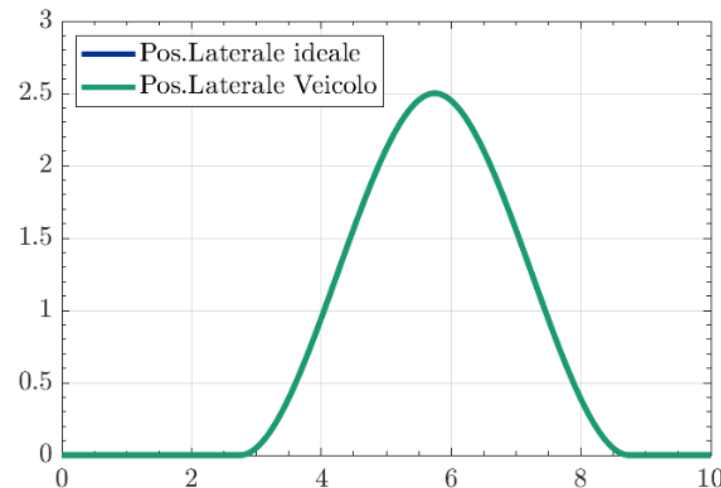
## Objective of the Lateral Dynamics Control

Find  $\delta_i(t) = \delta_i(u_i, v_i, \dots)$   
to ensure tracking of  $y_{id}(t), \psi_d(t)$

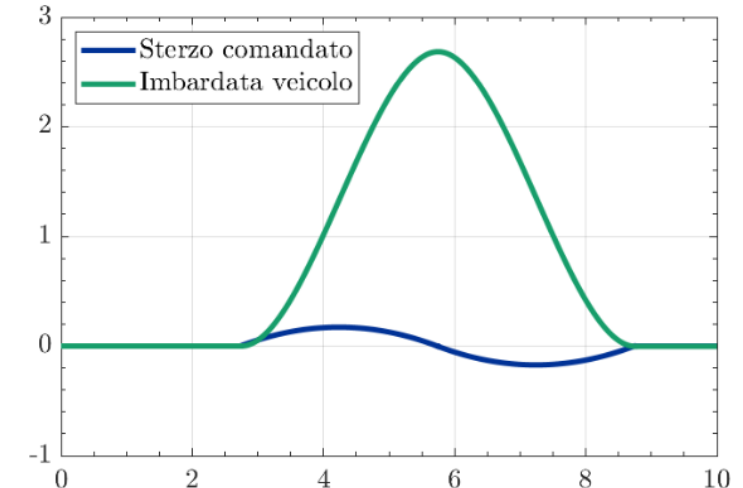
$$\eta_i = \begin{pmatrix} y_i - y_{id} \\ \psi_i - \psi_{id} \end{pmatrix}$$

$$\xi_i = \begin{pmatrix} v_i - \dot{y}_{id} \\ r_i - \dot{\psi}_{id} \end{pmatrix}$$

*i*th position & speed  
**tracking** errors



(a) Errore Posizione Laterale [m]



(b) Sterzo-Imbardata [gradi]





## Complete bicycle model, single track for generic path and vehicle configuration

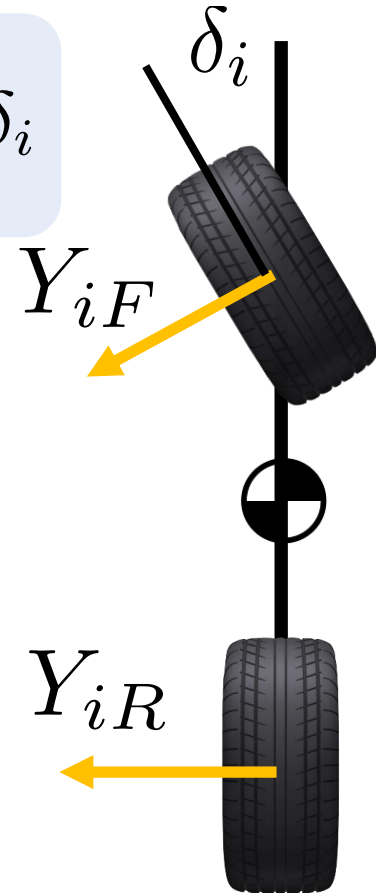
$$m_i (\dot{u}_i + v_i r_i) = X_i(u_i, \omega_i, t) - L_i(u_i) + C_{i1} \left( \frac{v_i + a_{iF} r_i}{u_i} - \delta_i \right) \delta_i$$

$$m_i (\dot{v}_i + u_i r_i) = Y_{iF}(\delta_i) + Y_{iR}(\delta_i)$$

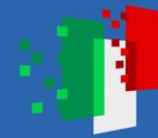
$$I_i \dot{r}_i = a_{iF} Y_{iF}(\delta_i) - a_{iR} Y_{iR}(\delta_i)$$

$$Y_{ij}(\delta_i) = Y_{ij} \left( \tau_{ij} \delta_i - \frac{v_i + r_i a_{ij}}{u_i} \right)$$

$j \in \{F, R\}$







## In summary, the proposed model and controller

*i*th longitudinal dynamics

$$\dot{x}_i = u_i$$

$$\dot{u}_i = \frac{1}{m_i} (X_i(u_i, \omega_i, t) - L_i(u_i))$$

$$\dot{\omega}_i = \frac{1}{J_i} (T_i - R_i X_i(u_i, \omega_i, t))$$

*i*th lateral dynamics

$$\dot{y}_i = v_i$$

$$\dot{v}_i = -u_i r_i - \gamma_{i1} \frac{v_i}{u_i} + \gamma_{i2} \frac{r_i}{u_i} + \gamma_{i3} \delta_i$$

$$\dot{\psi}_i = r_i$$

$$\dot{r}_i = \gamma_{i4} \frac{v_i}{u_i} - \gamma_{i5} \frac{r_i}{v_i} + \gamma_{i6} \delta_i$$

*i*th driving torque

$$T_i = m_i^* R_i (\dot{u}_{id} - k \delta u_i) + R_i L_i(u_i)$$

*i*th steering angle

$$\delta_i = \delta_i(u_i, y_i, v_i, \psi_i, r_i, t)$$

estimated loss





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That's all for now,  
folks!



**FORESEEN**

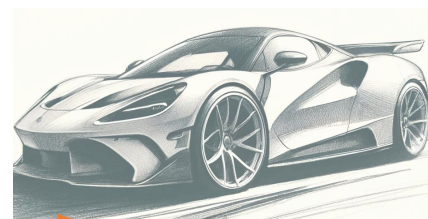
*i*th vehicle and  
*follower*



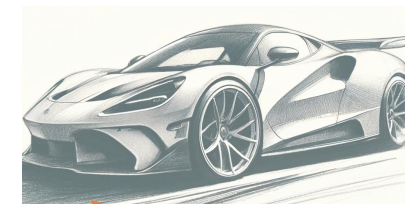
platoon  
*leader*

$A_0$

$u_{\text{leader}}$

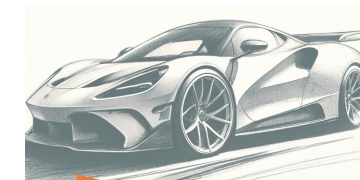


$d$



$d$

$A_{i-1}$



$d$

$A_i$

